Lexicographic products in metarouting

Alexander Gurney & Timothy Griffin
University of Cambridge

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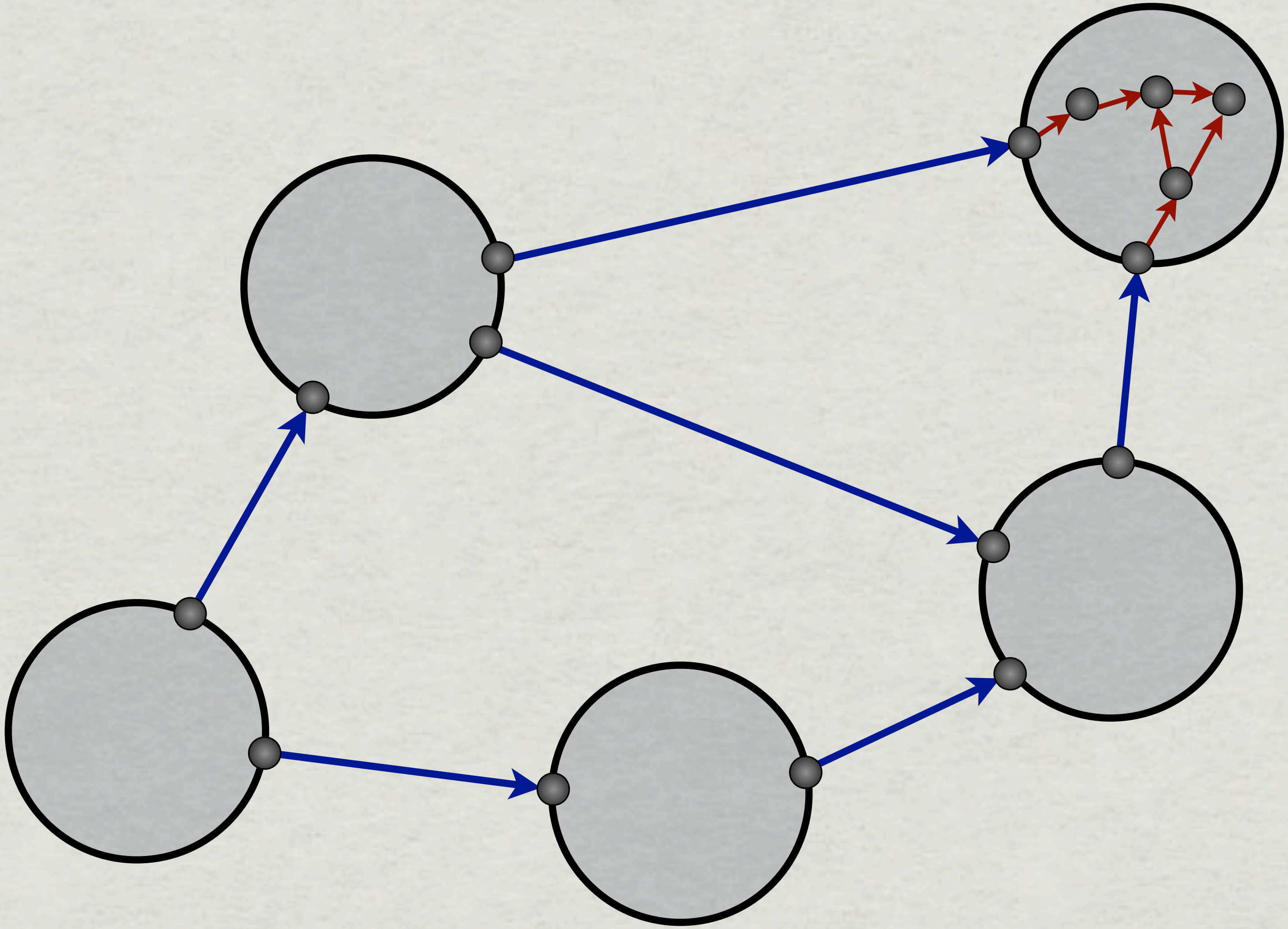
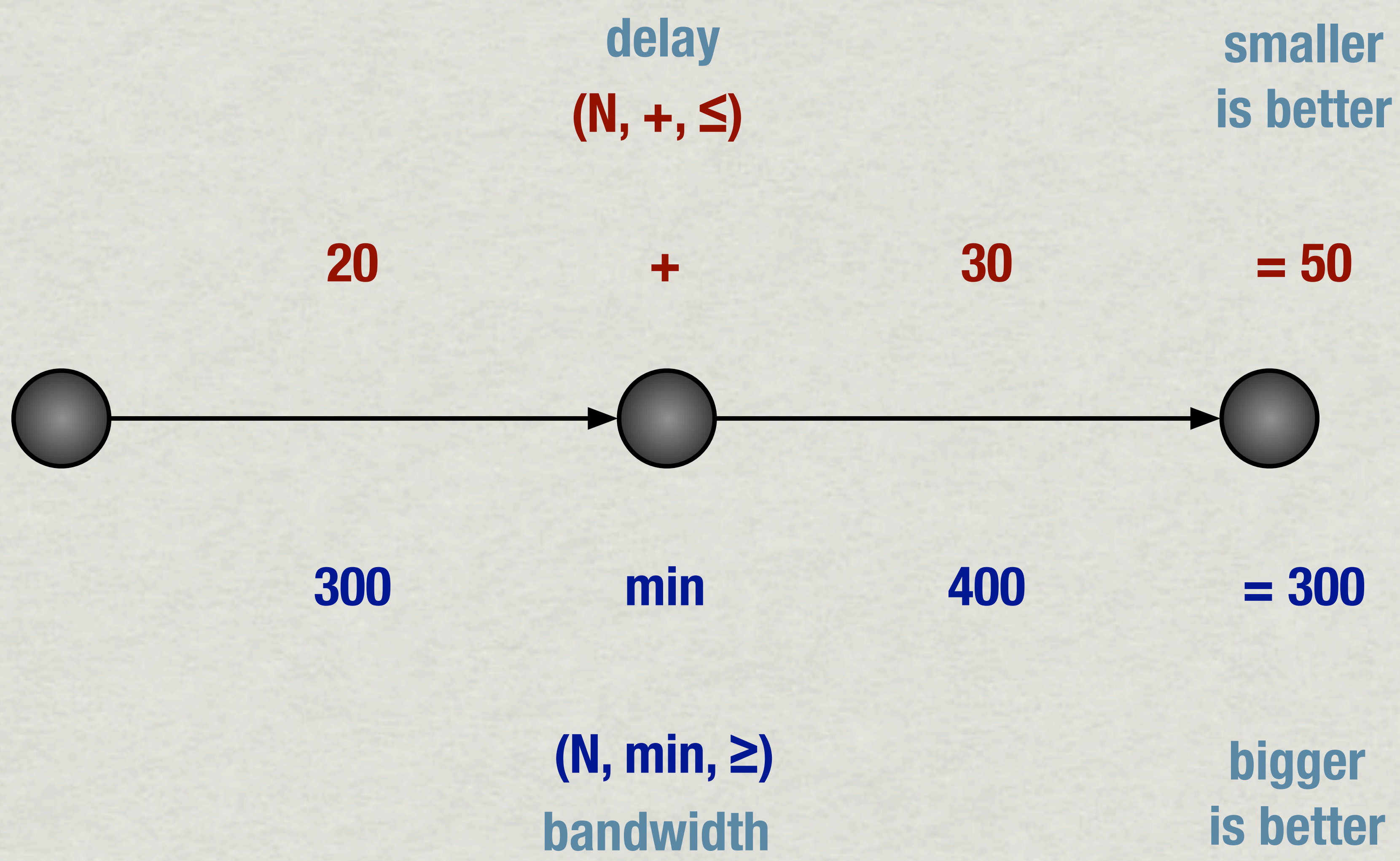
- Lexicographic choice is important (BGP, OSPF, ...)
- But it doesn't always work: not all metrics allow us to find best paths.
- When *precisely* can we find globally or locally optimum paths, using lexicographic choice?

lexicographic choice of **exterior metric** followed by **interior metric**





delay
(N, +, ≤)
smaller
is better
20
+
30
= 50
300
min
400
= 300
(N, min, ≥)
bandwidth
bigger
is better

$(S, \otimes, \leq)$

compute path weights
from link weights

$a \otimes b \otimes c \otimes d = \ldots$

compare path weights

$p \leq q$?

$p \sim q$ = same preference

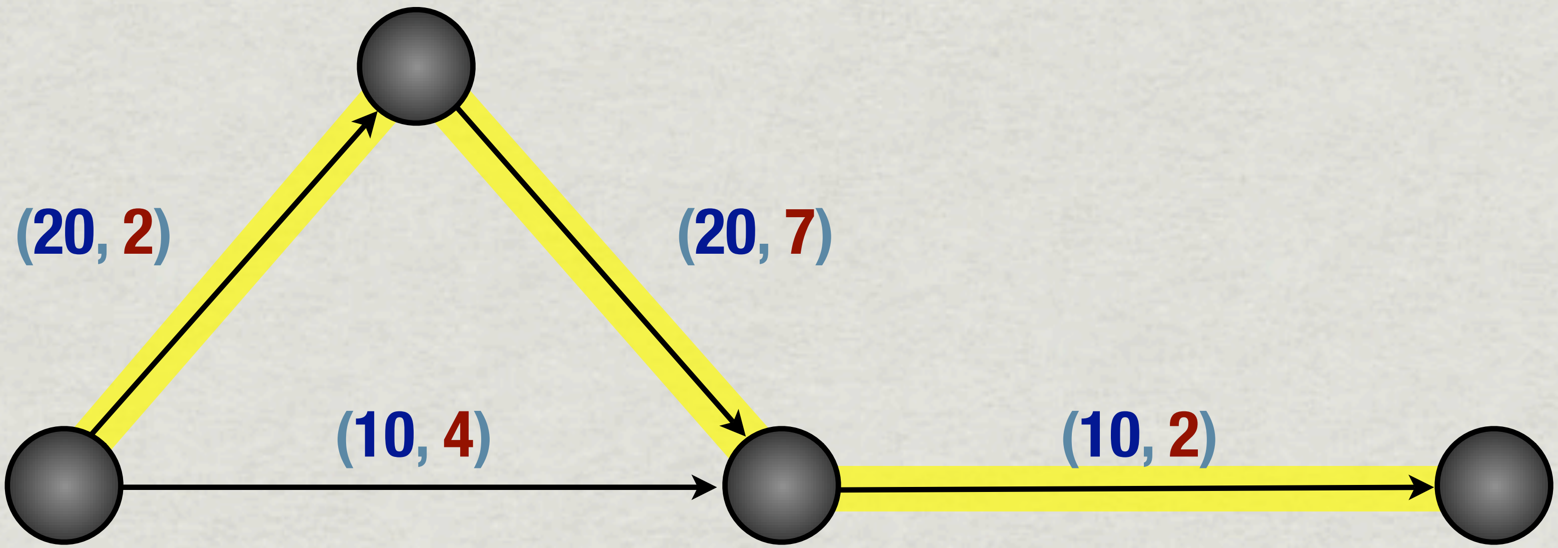
$p \# q$ = incomparable

or we could put functions on arcs
and compose them along a path

or we could have an operator
(like min, max) to choose the best path

that makes four possible families of structures

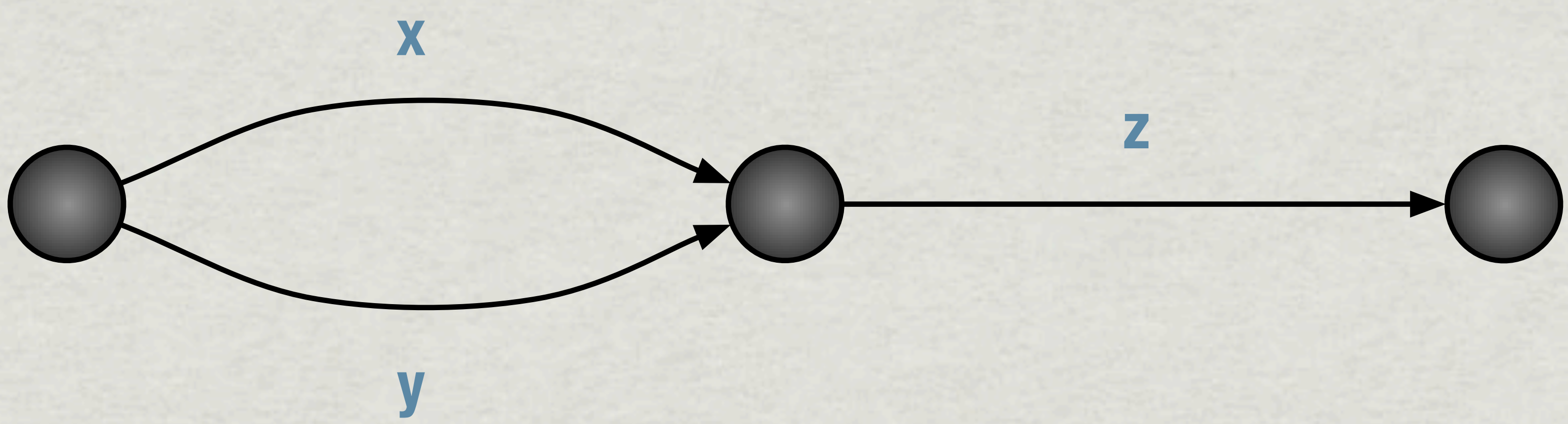
Bandwidth-delay fails



choose (20, 9)
over (10, 4)

have (10, 11)
and not (10, 6)

Monotonicity yields globally optimum paths



$$x \leq y \Rightarrow z \otimes x \leq z \otimes y$$

We need general rules

- bandwidth $\vec{\times}$ delay – not monotonic
- delay $\vec{\times}$ bandwidth – is monotonic
- hop-count $\vec{\times}$ delay $\vec{\times}$ reliability $\vec{\times}$ bandwidth – ???

MONOTONIC(S $\vec{\times}$ T) if and only if ...

lexicographic product

$$\mathrm{M}(S \vec{\times} T) \Leftrightarrow \mathrm{M}(S) \wedge \mathrm{M}(T) \wedge (\mathrm{N}(S) \vee \mathrm{C}(T))$$

$$z \otimes x = z \otimes y \Rightarrow x = y$$

“one-to-one” or “cancellative”

$$z \otimes x = z \otimes y$$

“constant”

- Proved for total orders: Tôru Saitô 1970

$$\mathrm{M}(S \overrightarrow{\times} T) \Leftrightarrow \mathrm{M}(S) \wedge \mathrm{M}(T) \wedge (\mathrm{N}(S) \vee \mathrm{C}(T))$$

$$z \otimes x \sim z \otimes y \Rightarrow x \sim y \vee x \# y$$

integers with addition

paths with concatenation

sets with disjoint union

probabilities with multiplication

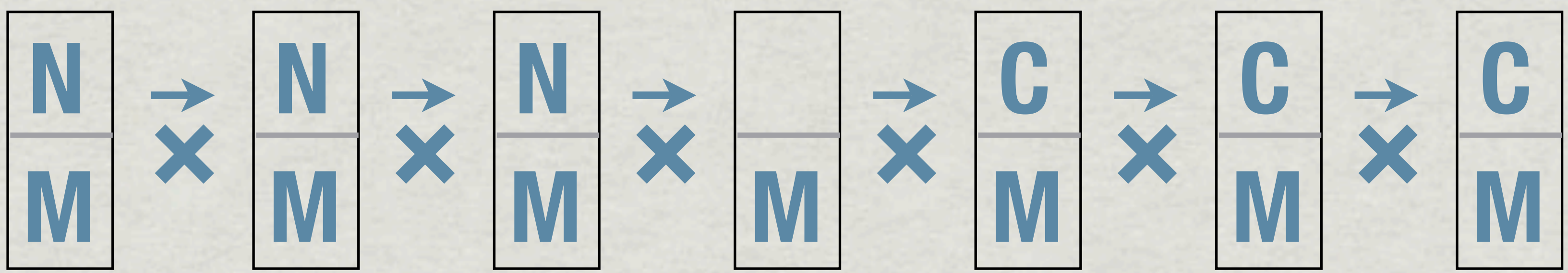
BUT NOT integers with min

$$z \otimes x \sim z \otimes y$$

choose left operand

“everything’s equivalent”

If it looks like this, then it's monotonic:



(need at least one N or at least one C)

delay $\overrightarrow{\times}$ reliability $\overrightarrow{\times}$ bandwidth $\overrightarrow{\times}$ constant **OK!**

delay $\overrightarrow{\times}$ bandwidth $\overrightarrow{\times}$ reliability **NOT OK**

reliability

$\vec{\times}$

delay

$\vec{\times}$

distinct areas

$\vec{\times}$

timestamp sequence

$0.98 \times 0.96 = 0.9408$
higher probabilities are better

M	N

$18 + 30 = 48$
lower delays are better

M	N

$\{15\} \cup \{12, 3, 100\} = \{15, 12, 3, 100\}$
$\{15\} \cup \{3, 15\} = \{3, 15\}$
use the subset order

M	

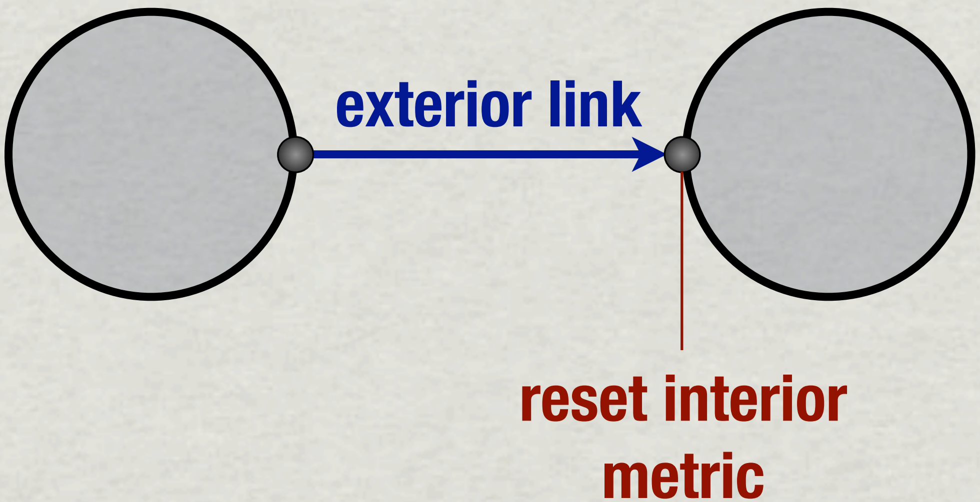
$08{:}22{:}01 \otimes [...] = [08{:}22{:}01, ...]$
everything is weighted the same

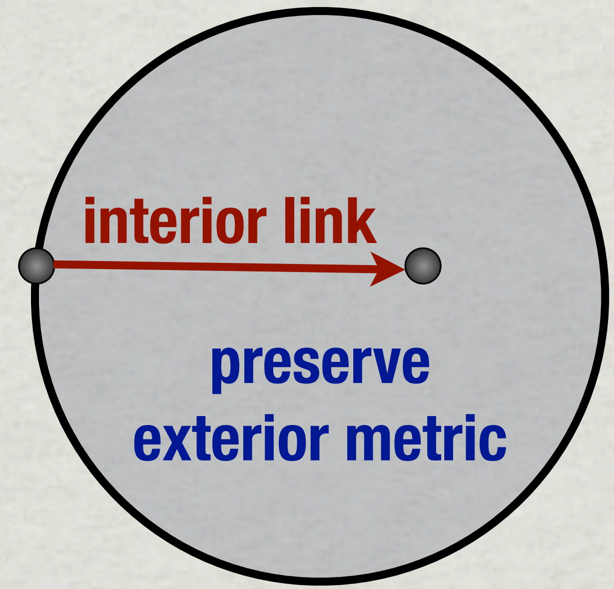
M	C

$\Downarrow$

OK, the whole product is monotonic

Scoped product: S ⊖ T





$$S \vec{\times} \mathrm{LEFT}(T) \quad + \quad \mathrm{RIGHT}(S) \vec{\times} T$$

$a \otimes b = a$ $\qquad$ $a \otimes b = b$

$$S \vec{\times} \mathrm{LEFT}(T) \quad + \quad \mathrm{RIGHT}(S) \vec{\times} T$$

$\mathrm{C}(\mathrm{LEFT}(T))$ always

$$z \otimes x = z \otimes y = z$$

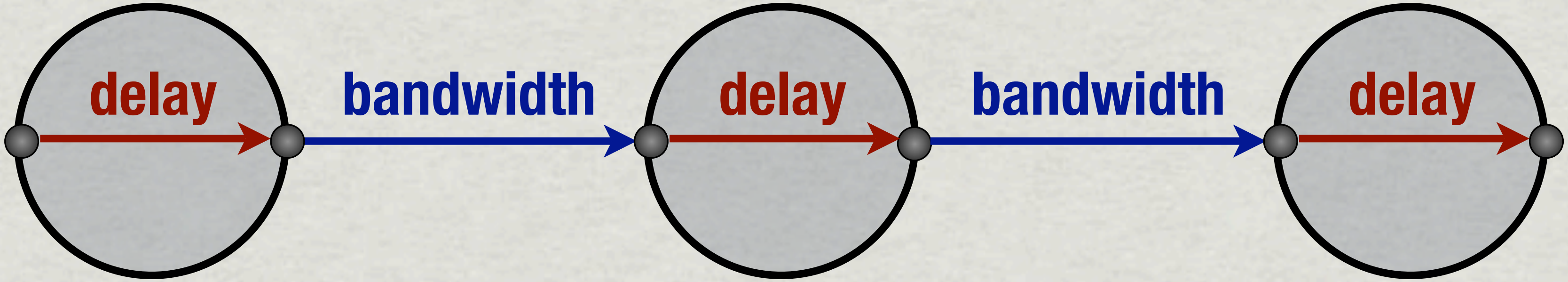
$\mathrm{N}(\mathrm{RIGHT}(S))$ always

$$z \otimes x = z \otimes y \Rightarrow x = y$$

$$\mathrm{M}(S \,\Theta\, T) \Leftrightarrow \mathrm{M}(S) \wedge \mathrm{M}(T)$$

$$\mathrm{M}(S \vec{\times} T) \Leftrightarrow \mathrm{M}(S) \wedge \mathrm{M}(T) \wedge (\mathrm{N}(S) \vee \mathrm{C}(T))$$

Bandwidth Θ delay: OK!



- The scoped product is *more permissive* than the ordinary lexicographic product
- We can find global optima this way

Local optimality

based on results by João Sobrinho

INCREASING

$x < z \otimes x$

$$\mathrm{I}(S \vec{\times} T) \Leftrightarrow \mathrm{I}(S) \vee (\mathrm{ND}(S) \wedge \mathrm{I}(T))$$

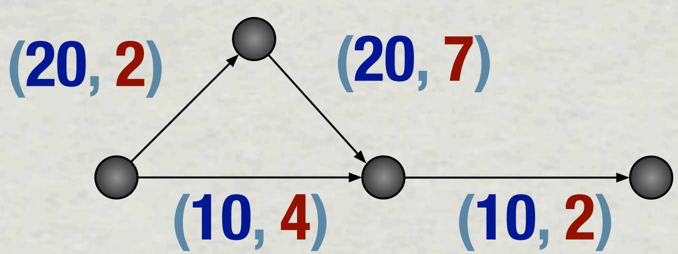
NONDECREASING

$x \leq z \otimes x$

$$\mathrm{ND}(S \vec{\times} T) \Leftrightarrow \mathrm{I}(S) \vee (\mathrm{ND}(S) \wedge \mathrm{ND}(T))$$

OK: $\boxed{\mathrm{ND}} \vec{\times} \boxed{\mathrm{ND}} \vec{\times} \boxed{\mathrm{ND}} \vec{\times} \boxed{\mathrm{I}} \vec{\times} \boxed{} \vec{\times} \boxed{} \vec{\times} \boxed{}$

(need at least one I)



- bandwidth-delay yields local optima
- just fine for path-vector

Summary

- Lexicographic choice is ubiquitous
- We have an easier way of telling whether or not a particular product will work
- This extends to more elaborate situations, like the scoped product
- Aids exploration of the routing metric design space

* What about other ways of combining metrics?
* EIGRP: use the formula “d + k/b” to compare
* How do we give structure to this technique?
* What properties do we need to track?

Metarouting aims to do all this and more

Thank you!
Any questions?